

Distinctive Features of Fuzzy Logic Control

Takeshi Yamakawa, Ph.D.

Founding Director, Fuzzy Logic Systems Institute (FLSI), Japan

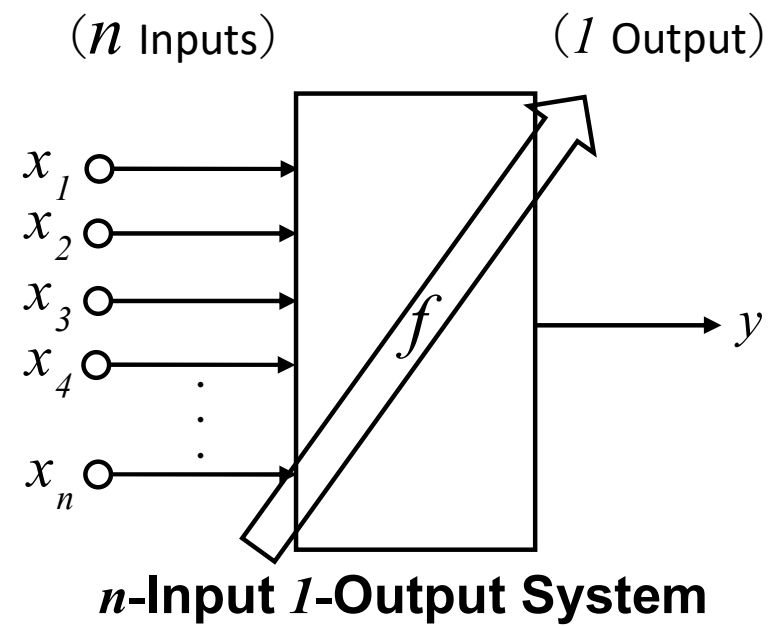
Professor Emeritus, Kyushu Institute of Technology, Japan

2008 IEEE CIS Fuzzy Systems Pioneer Awardee

Life Fellow of IEEE, IFSA, SOFT

October 19, 2019

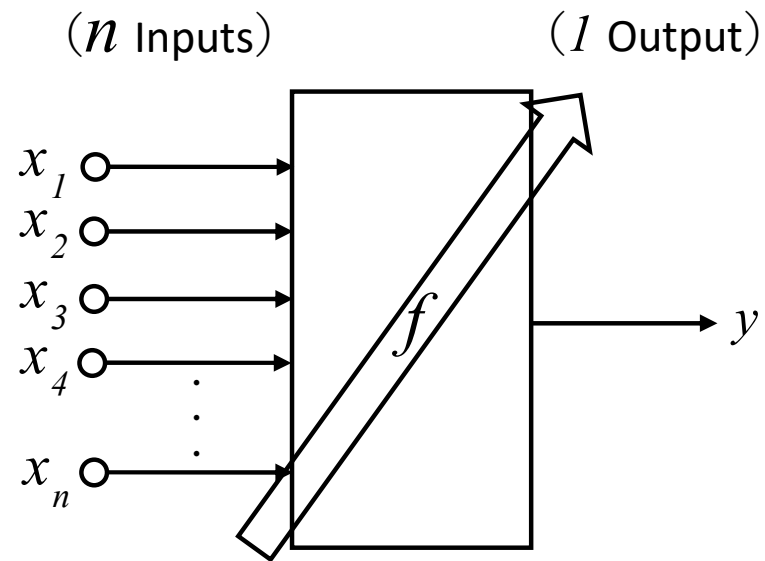
Kitakyushu, Japan



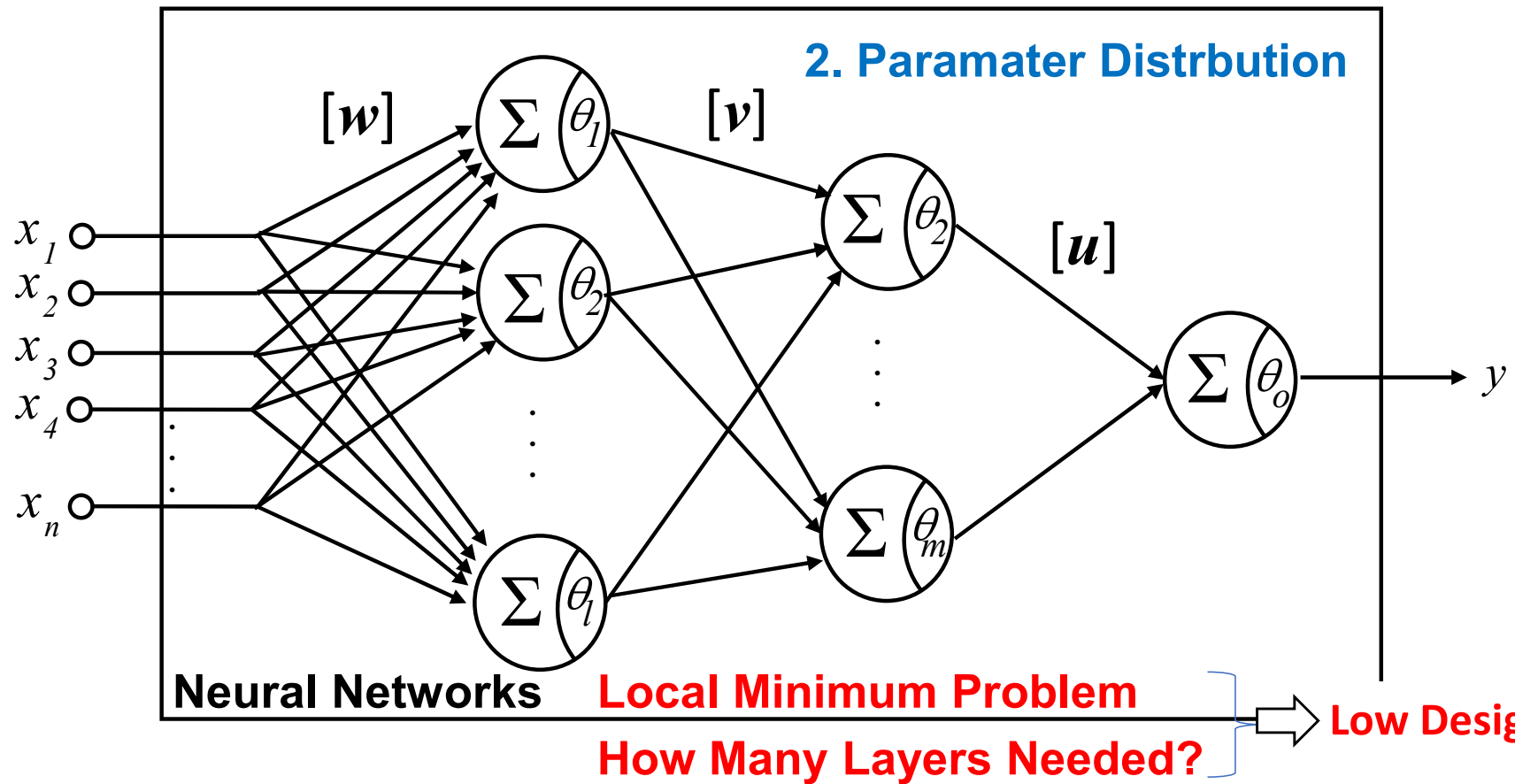
$$y = f(x_1, x_2, x_3, \dots, x_n)$$

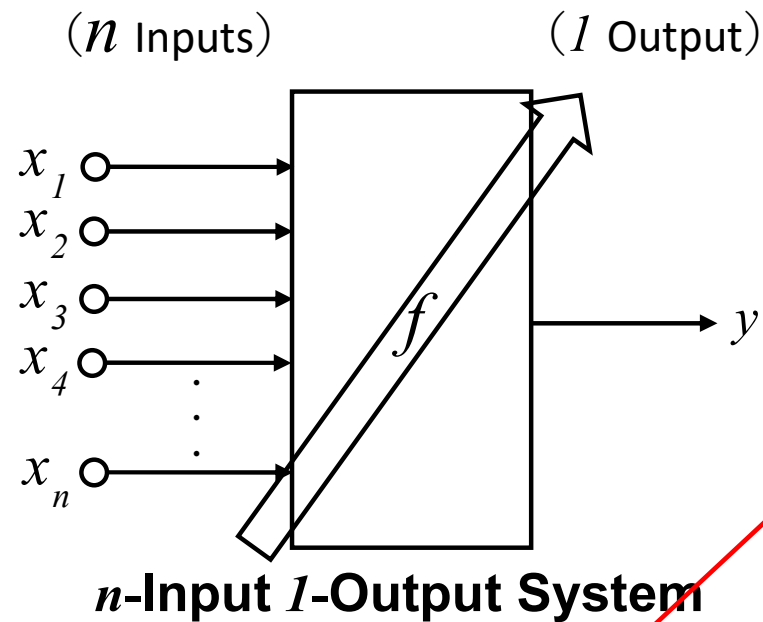
1. Mathematical Equation

Difficult to remodel



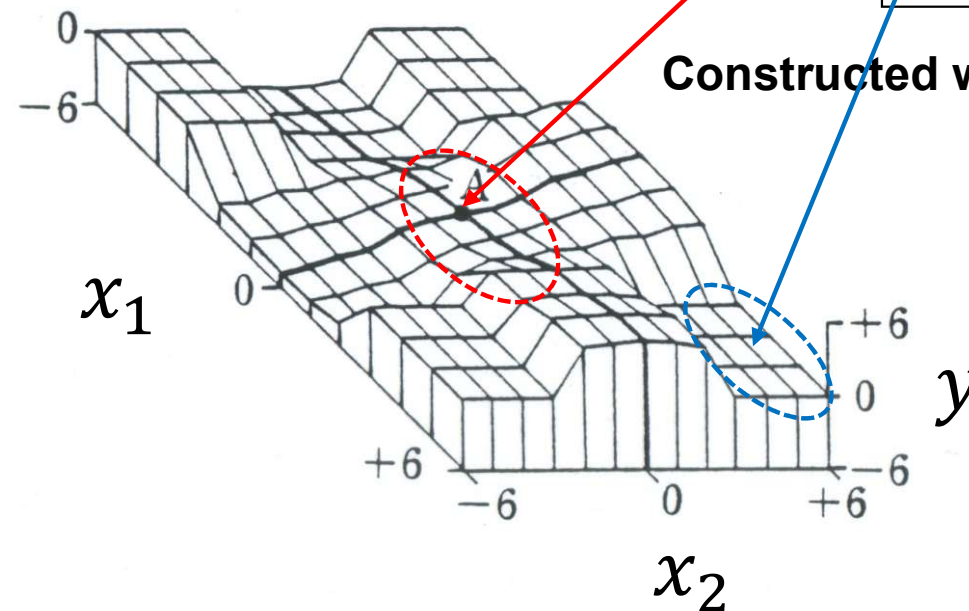
n -Input 1-Output System





If $x_1 \approx -6$ and $x_2 \approx -6$, then $y \approx 0$
 If $x_1 \approx -6$ and $x_2 \approx 0$, then $y \approx -6$
 If $x_1 \approx -6$ and $x_2 \approx +6$, then $y \approx 0$
 If $x_1 \approx 0$ and $x_2 \approx -6$, then $y \approx -6$
 If $x_1 \approx 0$ and $x_2 \approx 0$, then $y \approx 0$
 If $x_1 \approx 0$ and $x_2 \approx +6$, then $y \approx +6$
 If $x_1 \approx +6$ and $x_2 \approx -6$, then $y \approx 0$
 If $x_1 \approx +6$ and $x_2 \approx 0$, then $y \approx +6$
 If $x_1 \approx +6$ and $x_2 \approx +6$, then $y \approx 0$

Constructed with **9 Independent Rules**



Input-Output Characteristics of 2 Input-1 Output System

**High Describability
(Flexible)**

3. A Set of Fuzzy IF-THEN Rules + a Fuzzy Inference Engine + a Defuzzifier

Fuzzy Memory (Fuzzy IF-THEN Rules)

Rule NO.1 If x_1 is A_{11} and x_2 is A_{12} and x_3 is $A_{13} \cdot \cdot \cdot$, then y is B_1 .
Rule NO.2 If x_1 is A_{21} and x_2 is A_{22} and x_3 is $A_{23} \cdot \cdot \cdot$, then y is B_2 .
Rule NO.3 If x_1 is A_{31} and x_2 is A_{32} and x_3 is $A_{33} \cdot \cdot \cdot$, then y is B_3 .
.....
Rule NO.r If x_1 is A_{r1} and x_2 is A_{r2} and x_3 is $A_{r3} \cdot \cdot \cdot$, then y is B_r .

Label

Shape of Membership
Function

(n Inputs)

x_1 ○
 x_2 ○
 x_3 ○
 x_4 ○
⋮
 x_n ○

Fuzzy Inference Engine

$\overrightarrow{B'}$

**Defuzzifier
(Center of Gravity)**

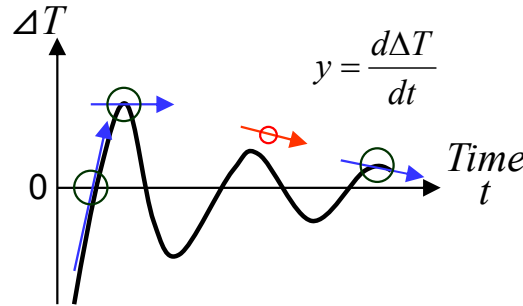
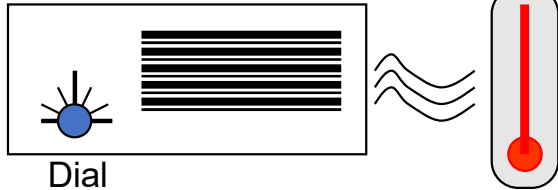
(1 Output)

y

n -Input 1 -Output Fuzzy System

Fuzzy Inference

Air Conditioning



Fuzzy IF-THEN Rules

- Rule1: If ΔT is **ZR** and y is **PL**, then z is **NL**.
 Rule2: If ΔT is **PL** and y is **ZR**, then z is **NM**.
 Rule3: If ΔT is **PS** and y is **NS**, then z is **ZR**.

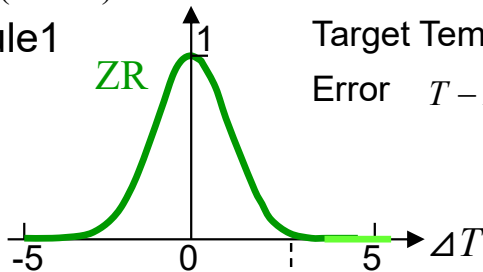
Fact : ΔT is **+2.5°C** and y is **-1°C/min**

Conclusion :

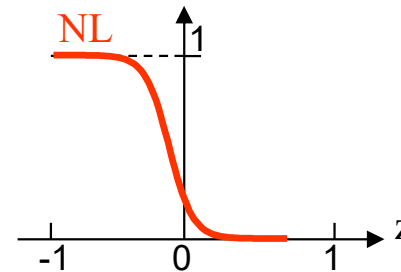
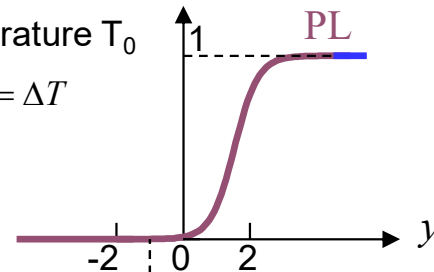
z is ??

PL: Positively Large
PM: Positively Medium
PS: Positively Small
ZR: Approximately Zero
NS: Negatively Small
NM: Negatively Medium
NL: Negatively Large

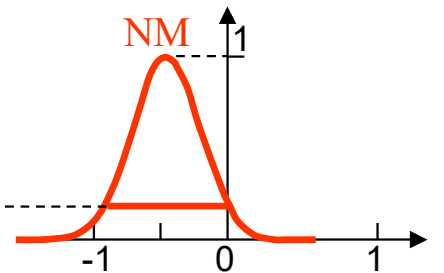
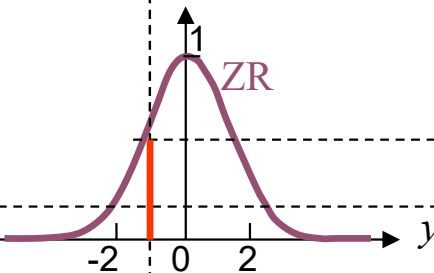
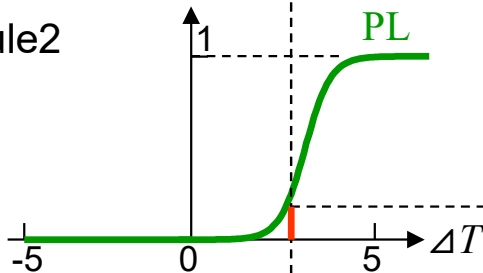
Rule1



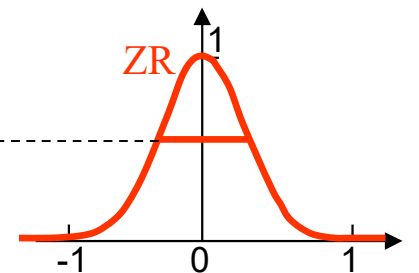
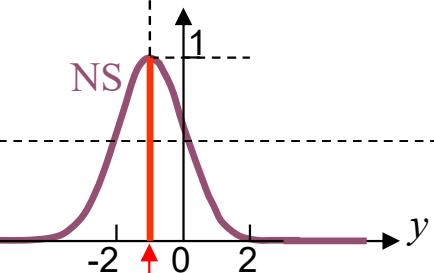
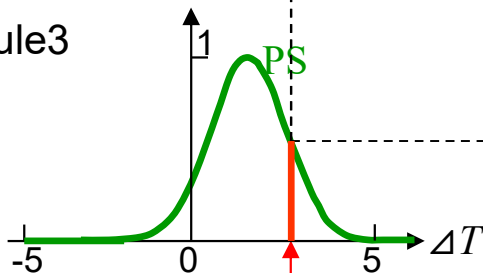
Room Temperature T
 Target Temperature T_0
 Error $T - T_0 = \Delta T$



Rule2



Rule3



-0.2
 Center of Gravity
 (Weighted Average)

$$z = \frac{\sum_{i=1}^n S_i C_i}{\sum_{i=1}^n S_i}$$

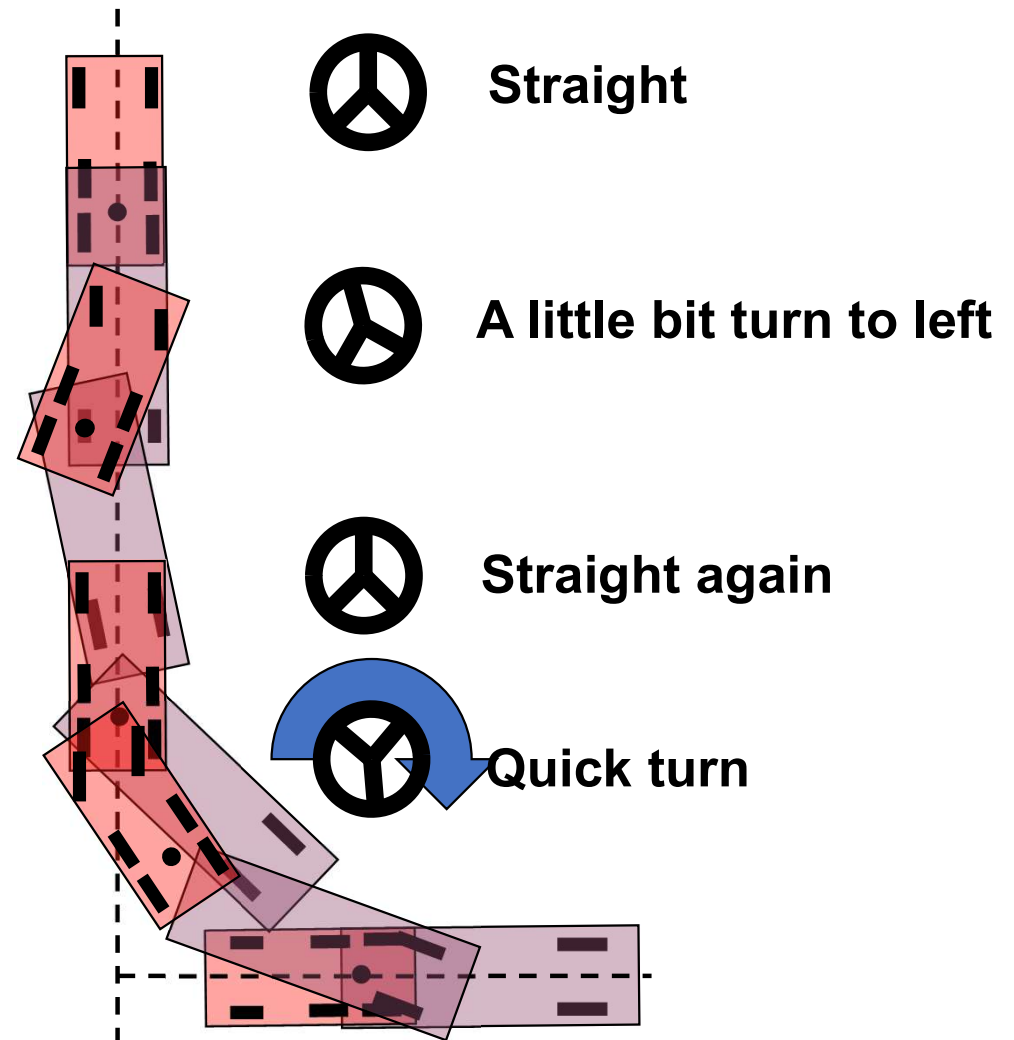
Knowledge Acquisition in the form of Fuzzy IF-THEN Rules and Execution of Fuzzy Inference —Trailer Truck Back-Up Control—



Trailer-Truck Back-Up Control

Difficulty in Trailer-Truck Back-Up

★ **Steering Wheel Control** ; opposite direction to ordinary cars



Trailer-Truck Back-Up Control

Difficulty in Trailer-Truck Back-Up

★ **Steering Wheel Control** ; opposite direction to ordinary cars

★ **Response of Trailer to Truck Movement** ; significant delay

Low speed → easy turning of steering wheel



difficult detection of timing for quick turn

High speed → easy detection of timing for



difficult turning of steering wheel



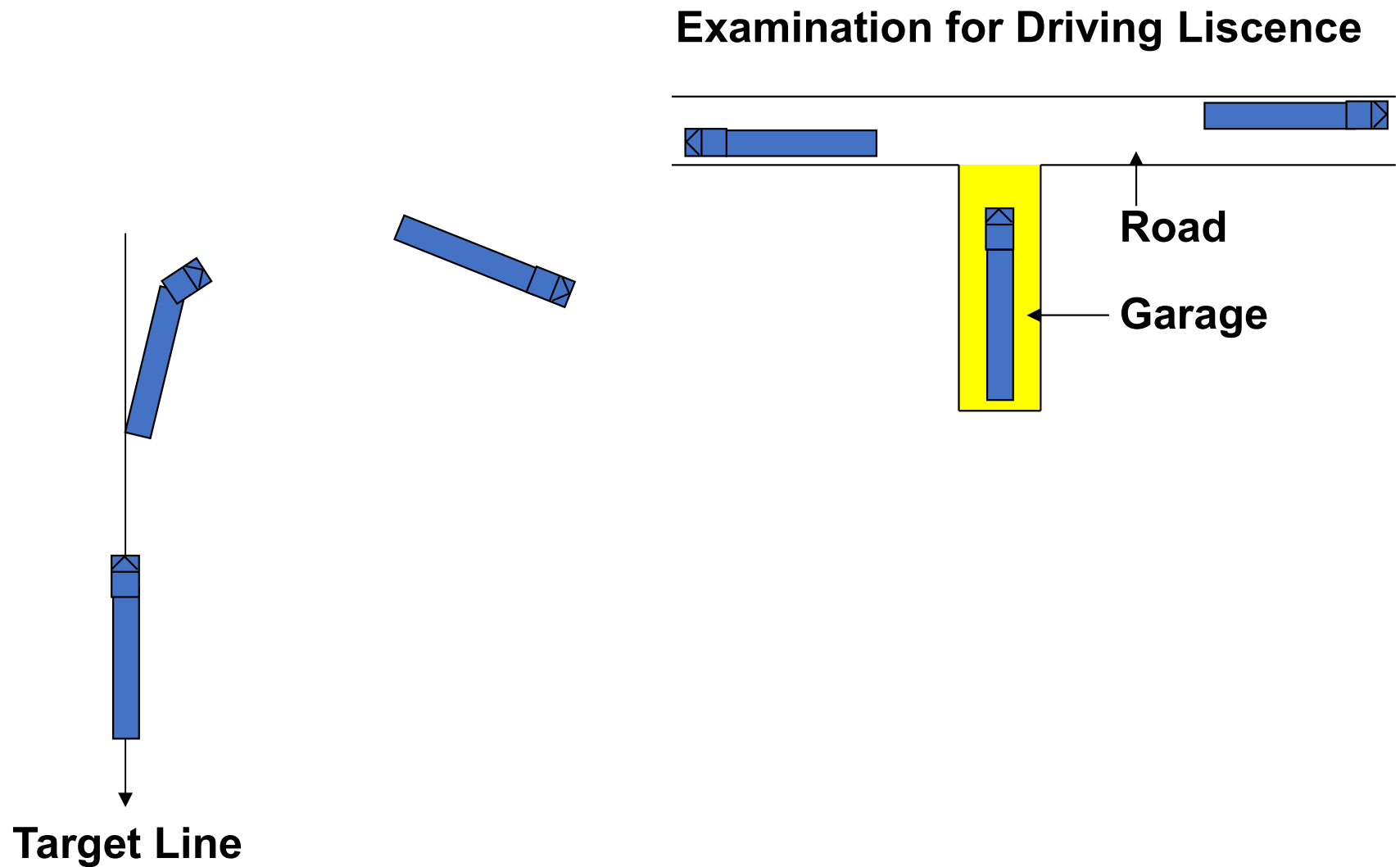
Semi-Trailer

★ **Clutch/Accelerator/Brake** ;
nonlinear action-effect

★ **Driving View** ; unsymmetrical

The uncontrollable state,
“Jackknife state”, must
be avoided.

Trailer-Truck Back-Up Control



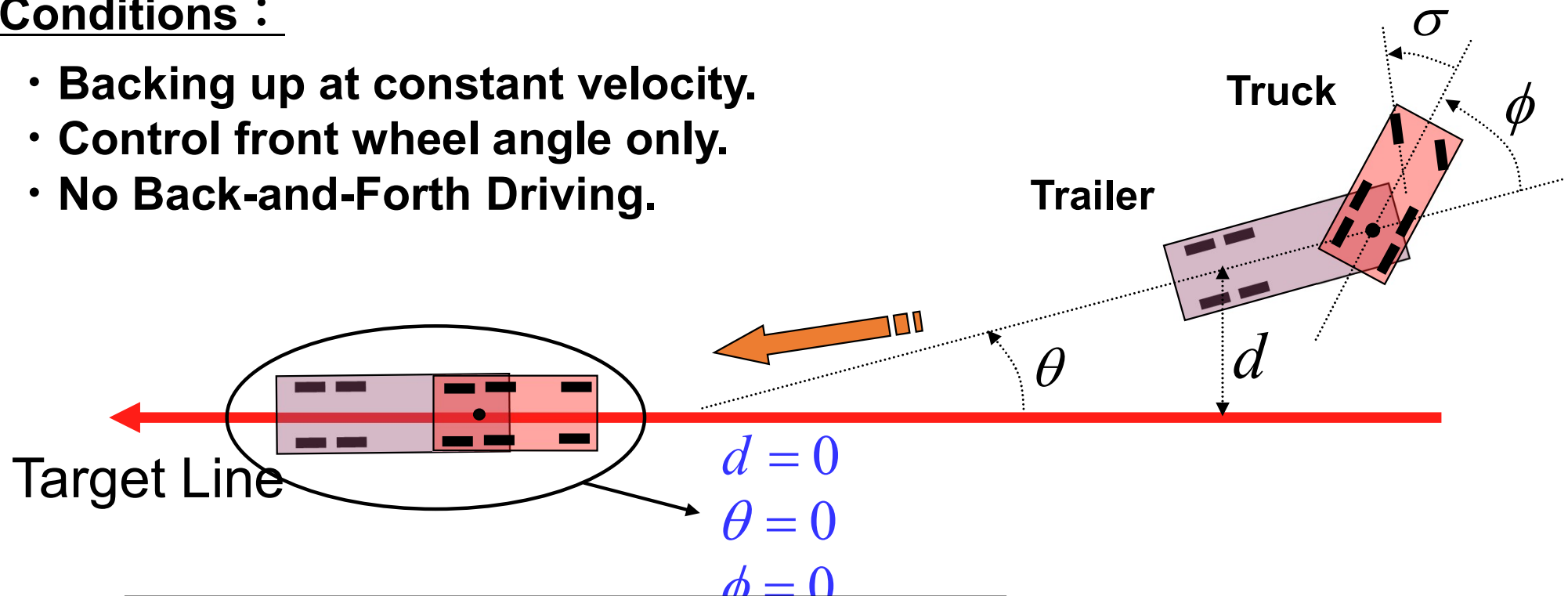
Trailer-Truck Back-Up Control

Control Objective :

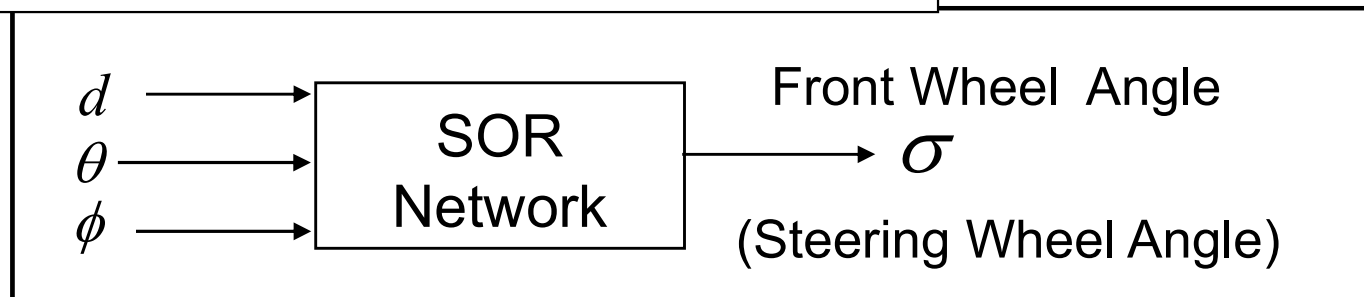
- Follow the target line from various initial state.

Conditions :

- Backing up at constant velocity.
- Control front wheel angle only.
- No Back-and-Forth Driving.



Design of 3-input 1-output controller



Design and Operation of a Controller for Trailer–Truck Back–Up

1. Generating learning vectors and evaluation



Difficult for Human Beings

Sampling of learning vectors (at random)

Evaluation of learning vectors

Proposed Method

Fuzzy Inference based Evaluation

2. Learning mode of SOR network

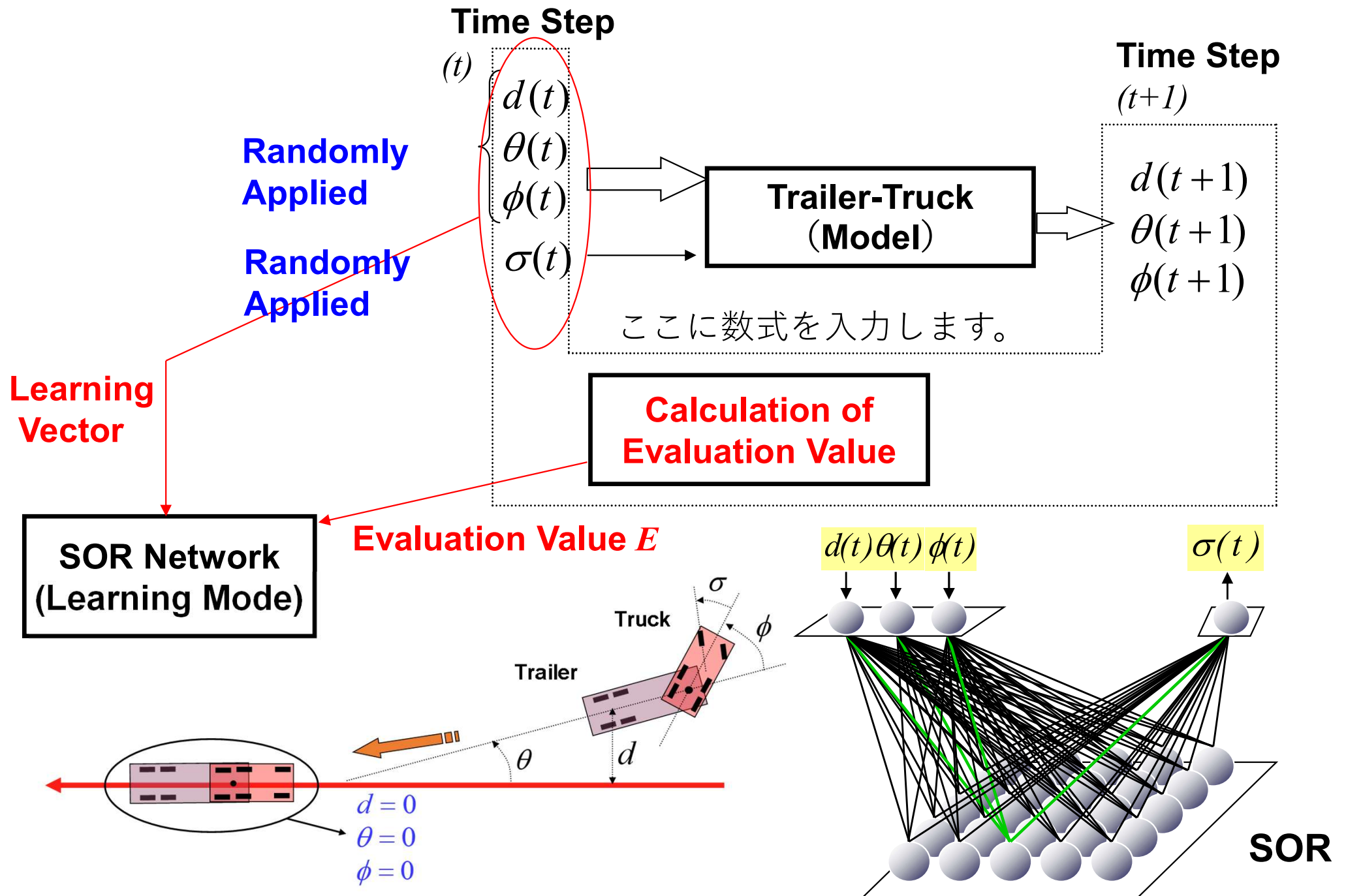
(The controller is automatically established by unsupervised learning)

3. Execution mode of SOR network

(The controller produces the adequate output in accordance with the input signals)

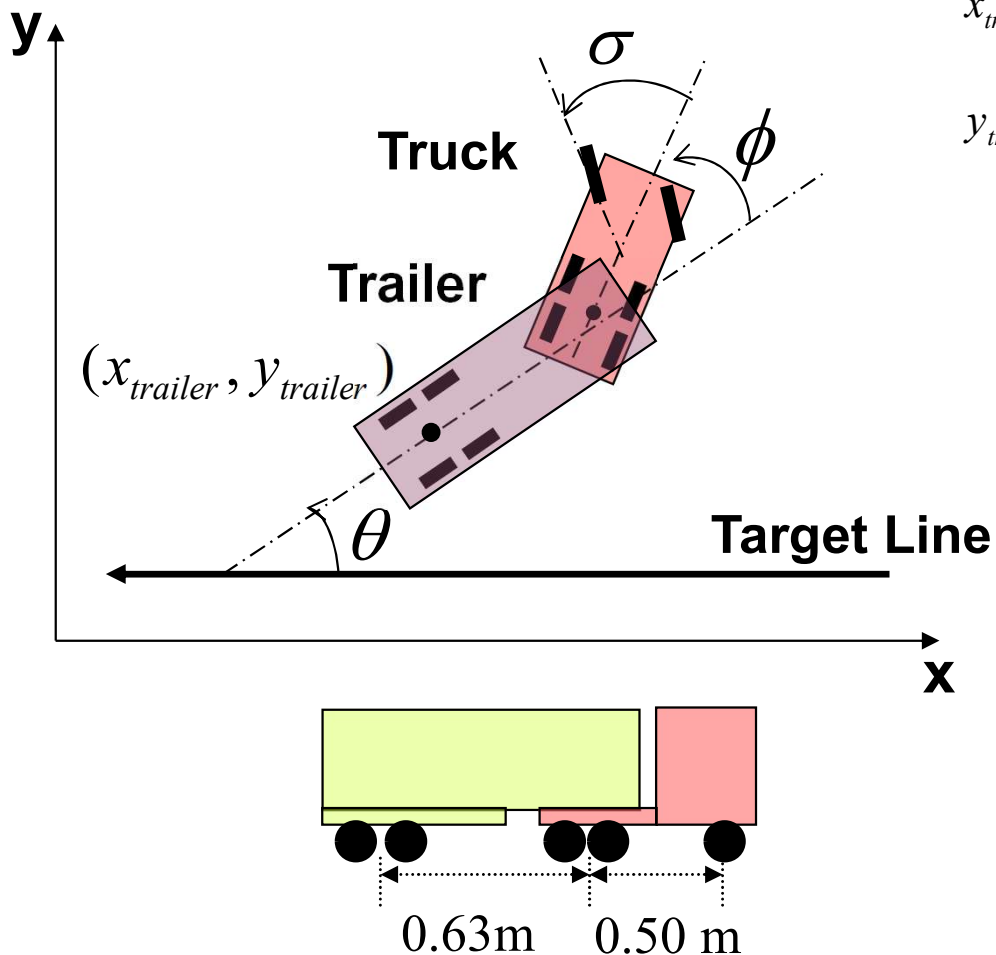
Learning Data Acquisition and Evaluation

SOR (Self-Organizing Relationship) Network



Kinematic Model of Semi-Trailer

- Mass, inertia, and friction, etc. are neglected.
- No sideslip of tires is assumed.



$$\theta_{truck}[i+1] = \theta_{truck}[i] + \frac{v \cdot \Delta t \cdot \tan \sigma[i]}{L_{truck}}$$

$$\theta_{trailer}[i+1] = \theta_{trailer}[i] + \frac{v \cdot \Delta t \cdot \sin \phi[i]}{L_{trailer}}$$

$$\phi[i] = \theta_{truck}[i] - \theta_{trailer}[i]$$

$$x_{trailer}[i+1] = x_{trailer}[i] + v \cdot \Delta t \cdot \cos \phi[i] \cdot \cos \frac{\theta_{trailer}[i+1] + \theta_{trailer}[i]}{2}$$

$$y_{trailer}[i+1] = y_{trailer}[i] + v \cdot \Delta t \cdot \cos \phi[i] \cdot \sin \frac{\theta_{trailer}[i+1] + \theta_{trailer}[i]}{2}$$

σ : Front wheel angle of truck (rad)

$\theta_{trailer}$: Trailer angle against target line(rad)

θ_{truck} : Truck angle against target line (rad)

ϕ : Angle between truck and trailer (rad)

$x_{trailer}$: x direction coordinate of trailer (m)

$y_{trailer}$: y direction coordinate of trailer (m)

L_{truck} : Wheel base of truck (m)

$L_{trailer}$: Wheel base of trailer (m)

Δt : Sampling interval (sec)

v : Backing up velocity (m/sec)

Evaluation by Fuzzy Inference

Antecedent Part of Fuzzy If-Then Rules

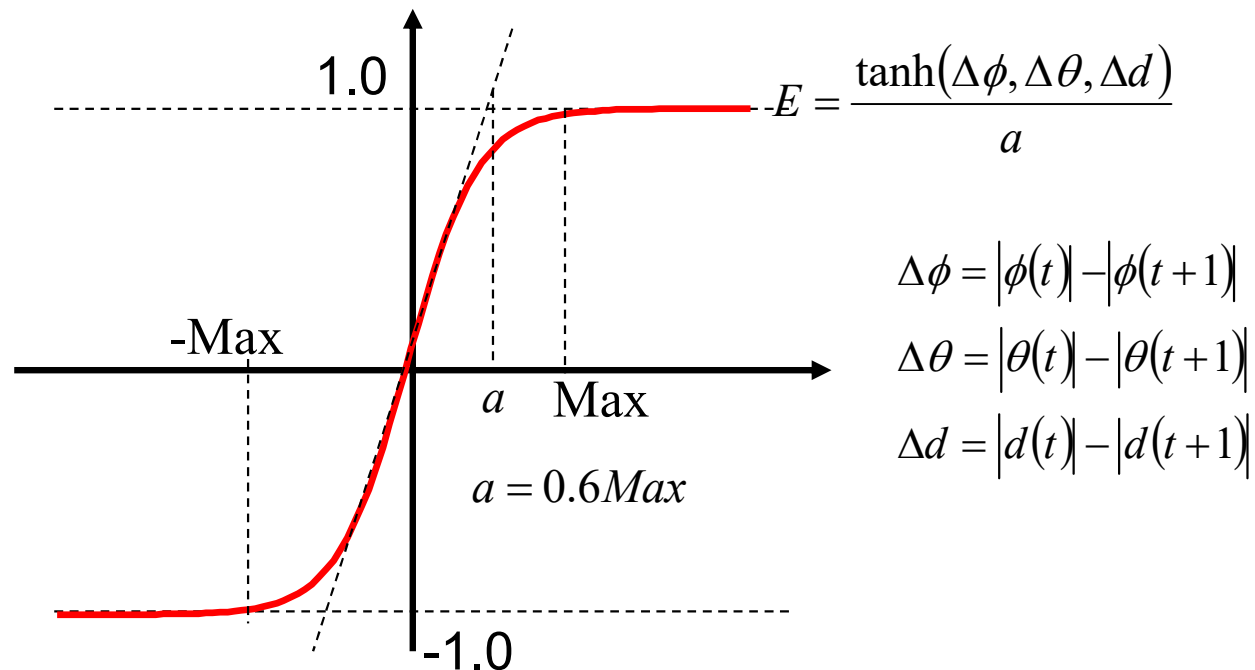
State of Trailer-Truck at t $(\phi(t), \theta(t), d(t))$.

Consequent Part of Fuzzy If-Then Rules.

Evaluation Value for I/O relationship of target system.

(Evaluation value is calculated based on **decrease of error** between t and $t+1$)

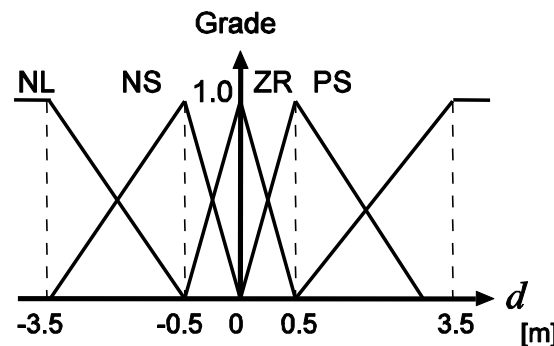
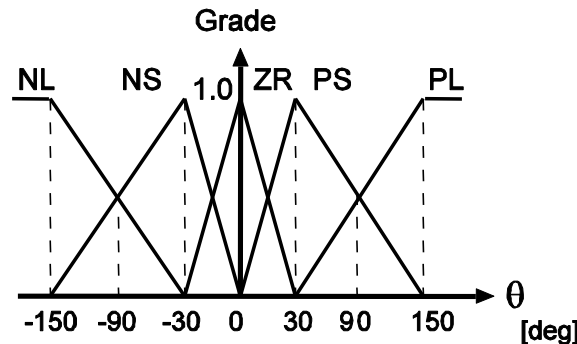
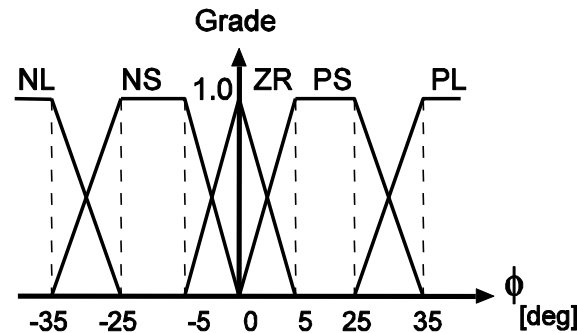
Evaluation Value $E \{E_\phi, E_\theta, E_d\}$



Evaluation by Fuzzy Inference

If ϕ is PS, θ is ZR and d is PL, then Evaluation is calculated by E_ϕ

				θ					
				NL	NS	ZR	PS	PL	
ϕ	PL	d	PL	$E\phi$					
			PS						
			ZR						
			NS						
			NL						
	PS	d	PL	$E\phi$			$\overline{Ed}$	$E\theta$	
			PS			$\overline{E\theta}$			
			ZR	$E\phi$		$E\theta$			
			NS					$\overline{E\phi}$	
			NL					$E\phi$	$\overline{Ed}$
	ZR	d	PL	$E\theta$		$\overline{Ed}$	$\overline{E\theta}$	$E\theta$	
			PS			$\overline{E\phi}$			
			ZR	$E\theta$		$E\theta$			
			NS					$\overline{E\theta}$	Ed
			NL					$E\theta$	
	NS	d	PL	$E\theta$		$\overline{Ed}$		$E\phi$	
			PS			$\overline{E\phi}$			
			ZR	$E\phi$					
			NS			$\overline{E\theta}$			
			NL			$E\theta$	$\overline{Ed}$		
	NL	d	PL	$E\phi$					
			PS						
			ZR						
			NS						
			NL						



If-Then Rule Table and Membership Function

Number of Rules: 5X5X5=125

The rule table means that which variable should be decreased in each situation.

The rules can be merged according to the situation as shown.

The rule table is symmetrical about a point.

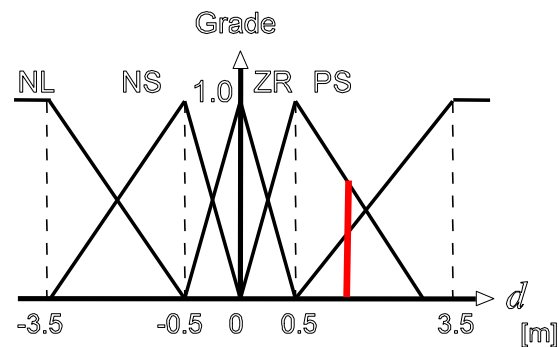
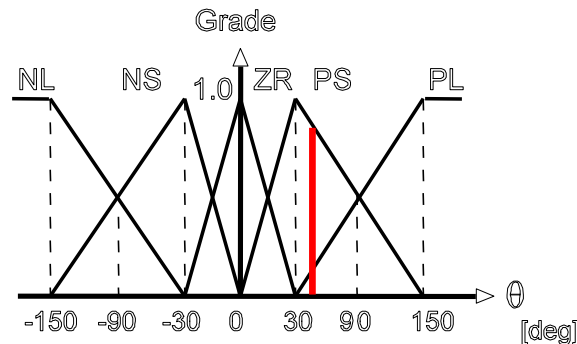
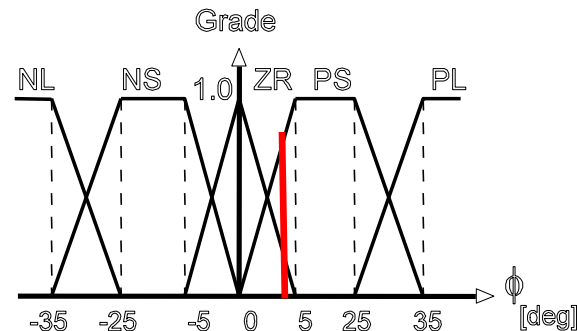


Practically, about 16 rules should be considered.

Evaluation by Fuzzy Inference

Complementary Membership Function → 2X2X2=8 rules activated

				θ				
				NL	NS	ZR	PS	PL
ϕ	PL	d	PL	$E\phi$				
			PS					
			ZR					
			NS					
			NL					
	PS	d	PL	$E\phi$		E_d		$E\theta$
			PS					
			ZR	$E\phi$		E_d		$E\theta$
			NS					
			NL					
	ZR	d	PL	$E\theta$		E_d		$E\theta$
			PS					
			ZR	$E\phi$		E_d		$E\theta$
			NS					
			NL					
	NS	d	PL	$E\theta$		E_d		$E\phi$
			PS					
			ZR	$E\phi$		E_d		$E\theta$
			NS					
			NL					
	NL	d	PL	$E\phi$				
			PS					
			ZR					
			NS					
			NL					



Example of Inference Process :

Fact :

ϕ is 4.0, θ is 45.0 and d is 1.0

Conclusion:

$$E = \frac{(\mu_1 + \mu_2)E_\phi + (\mu_3 + \mu_4)E_\theta + (\mu_5 + \mu_6 + \mu_7 + \mu_8)E_d}{\sum_{i=1}^8 \mu_i}$$

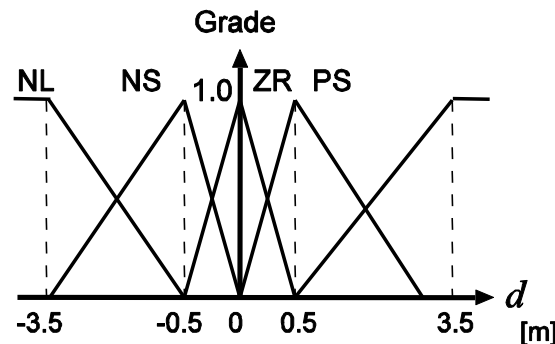
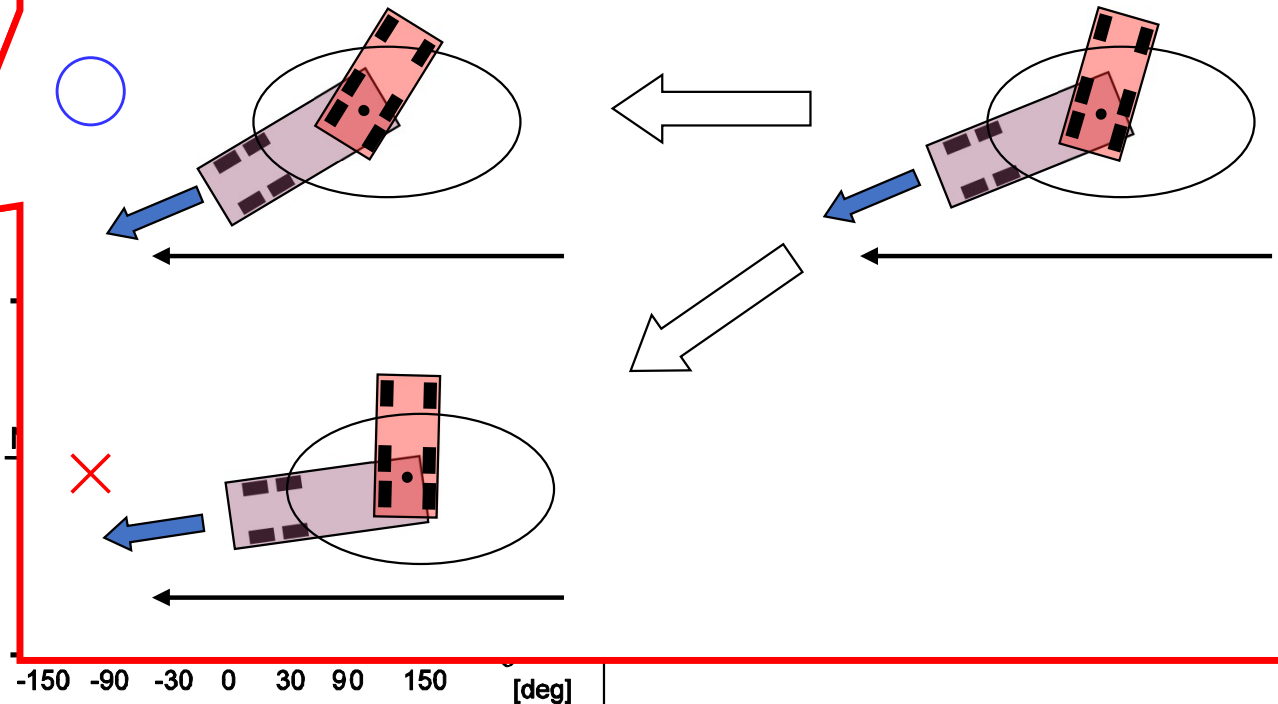
The Evaluation Value is a weighted average of the activated rules.

If-Then Rule Table and Membership Function

Evaluation by Fuzzy Inference

				θ					
				NL	NS	ZR	PS	PL	
ϕ	PL	d	PL	$E\phi$					
			PS						
			ZR						
			NS						
			NL						
	PS	d	PL	$E\phi$			Ed	$E\theta$	
			PS				$E\theta$		
			ZR				$E\theta$		
			NS	$E\phi$					
			NL	Ed					
	ZR	d	PL	$E\theta$		Ed	Ed	$E\theta$	
			PS			$E\theta$			
			ZR			$E\phi$	$E\theta$		
			NS	$E\theta$	$E\theta$				
			NL	Ed					
	NS	d	PL	$E\theta$		Ed	$E\phi$		
			PS			$E\phi$			
			ZR			$E\phi$			
			NS	$E\theta$					
			NL	Ed					
	NL	d	PL	$E\phi$					
			PS						
			ZR						
			NS						
			NL						

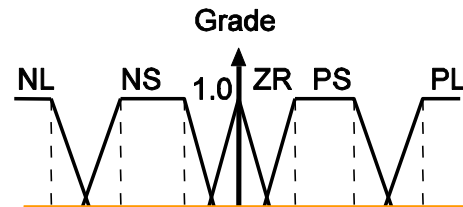
Avoiding Jack-knife State



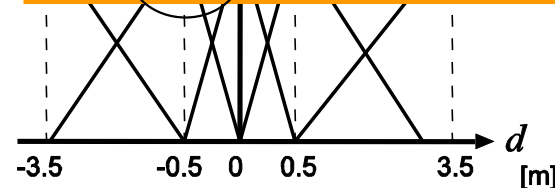
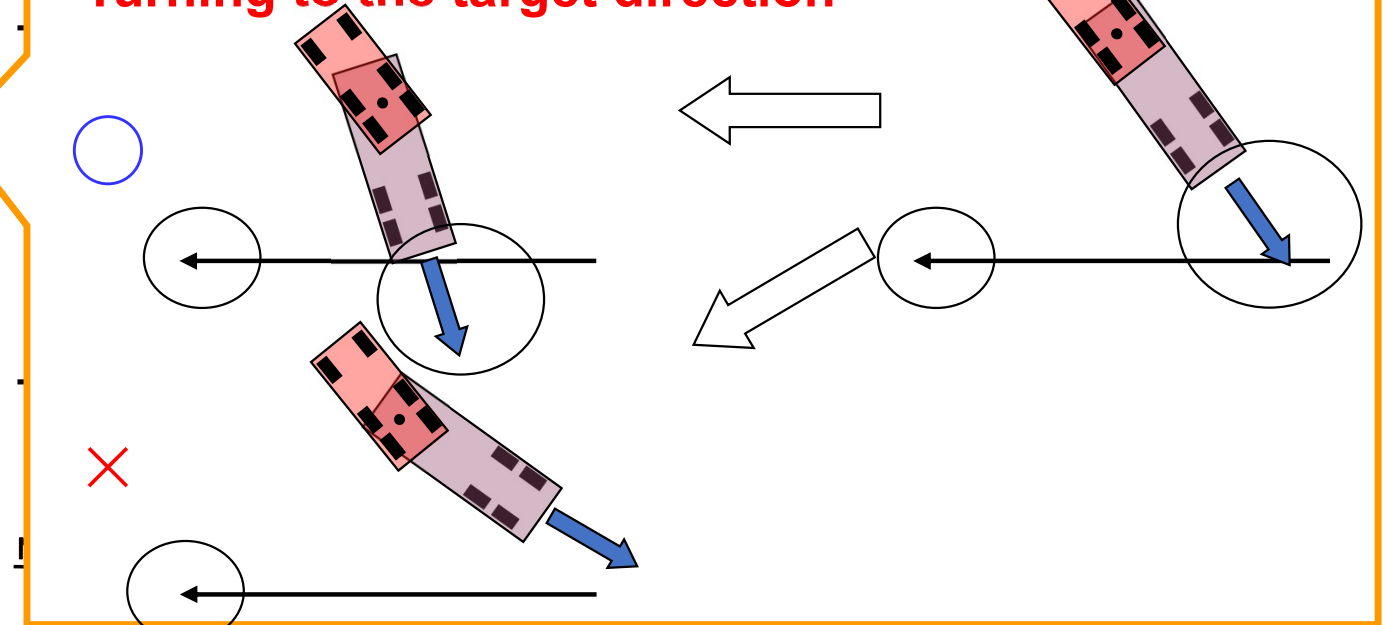
Rule Table and Membership Function

Evaluation by Fuzzy Inference

				θ						
				NL	NS	ZR	PS	PL		
ϕ	PL	d	PL	$E\phi$						
			PS							
			ZR							
			NS							
			NL							
	PS	d	PL	$E\phi$			Ed	$E\theta$	$E\theta$	
			PS				$E\theta$			
			ZR				$E\theta$			
			NS	$E\phi$						
			NL	$E\phi$	Ed					
	ZR	d	PL	$E\theta$		Ed	Ed	$E\theta$	$E\theta$	
			PS			$E\theta$				
			ZR	$E\theta$		$E\phi$	$E\theta$			
			NS			$E\theta$				
			NL			$E\theta$				Ed
	NS	d	PL	$E\theta$		Ed		$E\phi$		
			PS			$E\phi$				
			ZR	$E\phi$						
			NS				$E\theta$			
			NL				$E\theta$			Ed
	NL	d	PL	$E\phi$						
			PS							
			ZR							
			NS							
			NL							



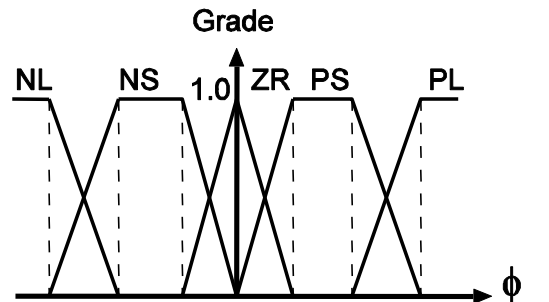
Turning to the target direction



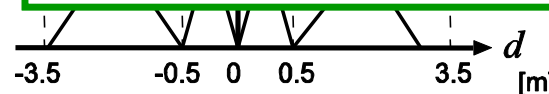
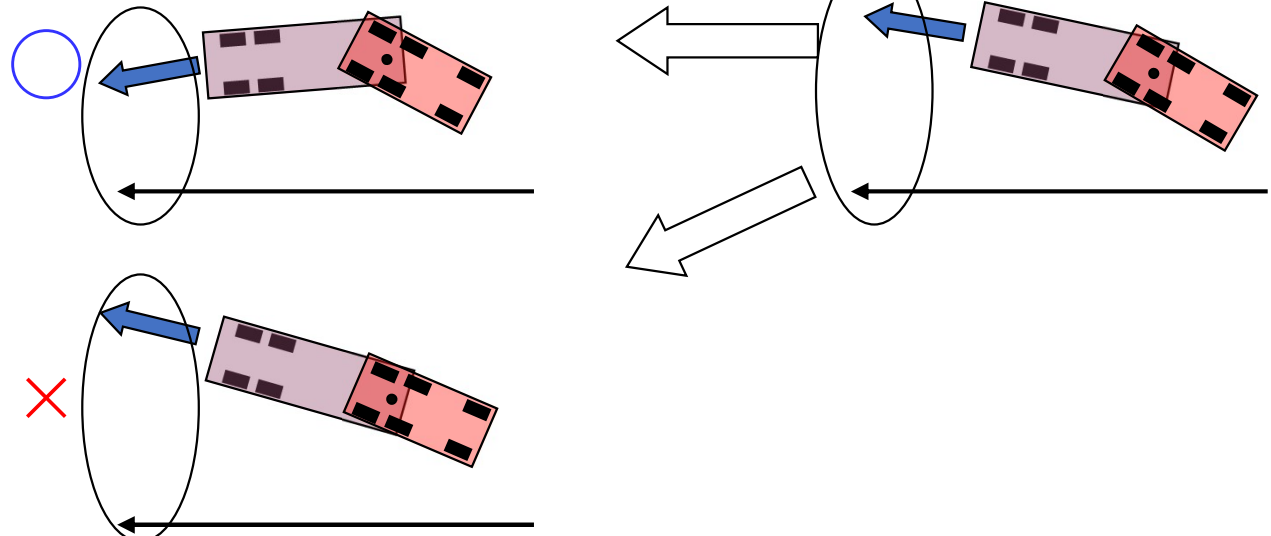
Rule Table and Membership Function

Evaluation by Fuzzy Inference

				θ						
				NL	NS	ZR	PS	PL		
ϕ	PL	d	PL	$E\phi$						
			PS							
			ZR							
			NS							
			NL							
	PS	d	PL	$E\phi$		Ed	$E\theta$			
			PS			$E\theta$				
			ZR	$E\phi$		$E\theta$				
			NS							
			NL							
	ZR	d	PL	$E\theta$		Ed	$E\theta$			
			PS			$E\theta$				
			ZR	$E\phi$		$E\phi$	$E\theta$			
			NS			$E\theta$				
			NL			$E\theta$			Ed	
	NS	d	PL	$E\theta$		Ed	$E\phi$			
			PS			$E\phi$				
			ZR	$E\phi$		$E\phi$				
			NS							
			NL						$E\theta$	Ed
	NL	d	PL	$E\phi$						
			PS							
			ZR							
			NS							
			NL							



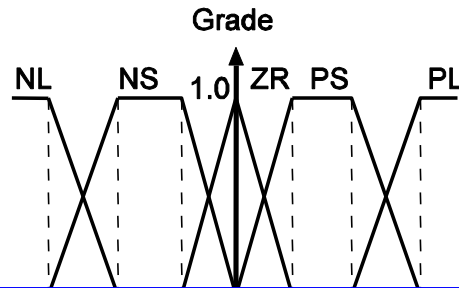
Turning toward the target line



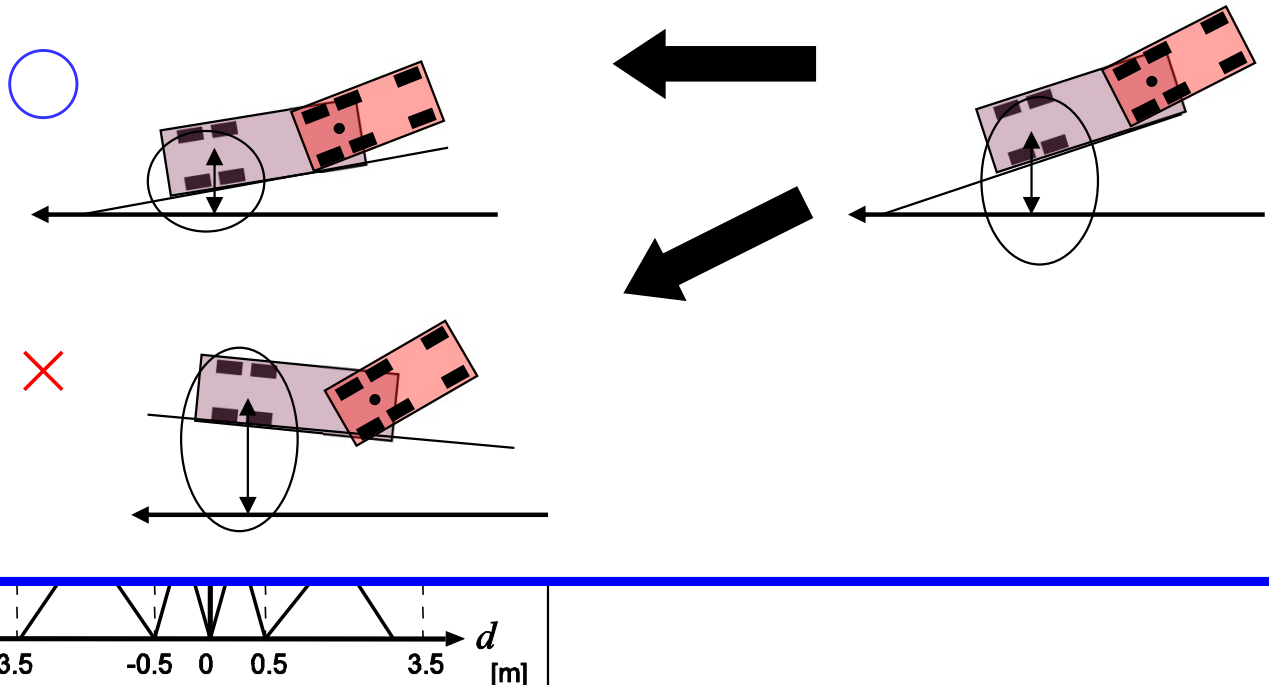
Rule Table and Membership Function

Evaluation by Fuzzy Inference

				θ					
				NL	NS	ZR	PS	PL	
ϕ	PL	d	PL	$E\phi$					
			PS						
			ZR						
			NS						
			NL						
	PS	d	PL	$E\phi$		Ed $E\theta$ $E\theta$			
			PS						
			ZR	$E\theta$		$E\theta$			
			NS						
			NL						
	ZR	d	PL	$E\theta$		Ed $E\phi$ $E\theta$			
			PS						
			ZR	$E\theta$		$E\theta$			
			NS						
			NL						
	NS	d	PL	$E\theta$		Ed		$E\phi$	
			PS						
			ZR	$E\phi$		$E\phi$			
			NS						
			NL						
	NL	d	PL	$E\phi$					
			PS						
			ZR						
			NS						
			NL						



Regulating angles and distance



Rule Table and Membership Function

Condition of Computer Simulation

SOR Network (Batch Learning)

Number of Learning Vector: $9^4=6,561$

Number of units: $25 \times 25=625$

Parameter of Neighborhood Function : $r \ 25 \rightarrow 0.25$

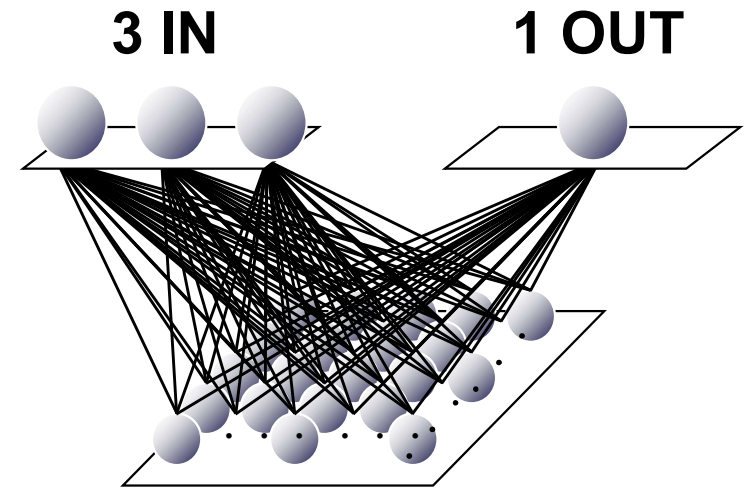
Value of Attractive Learning Rate : $\alpha \ 1.0 \rightarrow 0.01$

Value of Repulsive Learning Rate : $\beta \ 0.1 \rightarrow 0.005$

Radius of Repulsive Field: $r_r \ 0.1$

Learning Iteration: 200

Decay Rate: 30.0



$$\left. \begin{array}{l} a_{\phi} = 1.5^{\circ} \\ a_{\theta} = 1.9^{\circ} \\ a_d = 0.03m \end{array} \right\} = 0.6 \times (\phi_{\max}, \theta_{\max}, d_{\max})$$

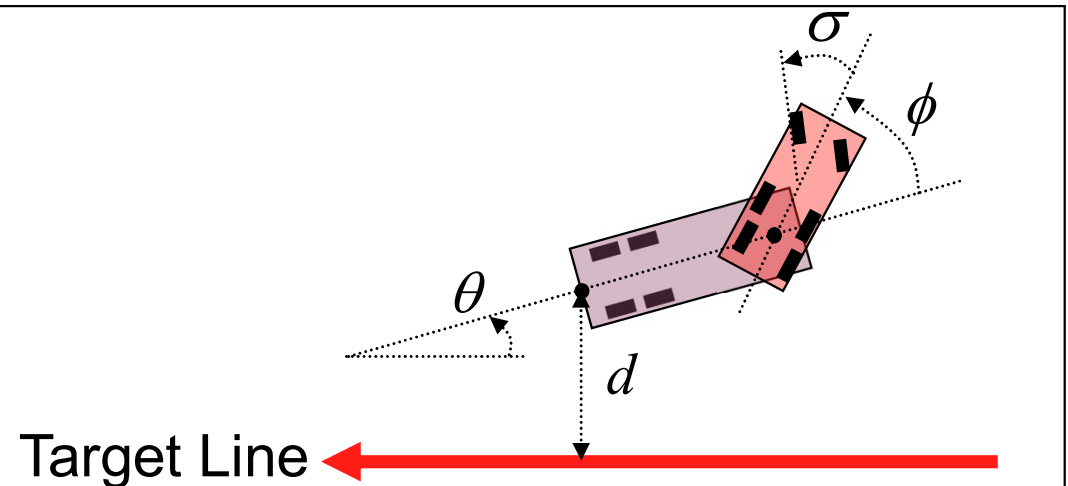
Range of Variables

$$-180^{\circ} \leq \theta \leq 180^{\circ}$$

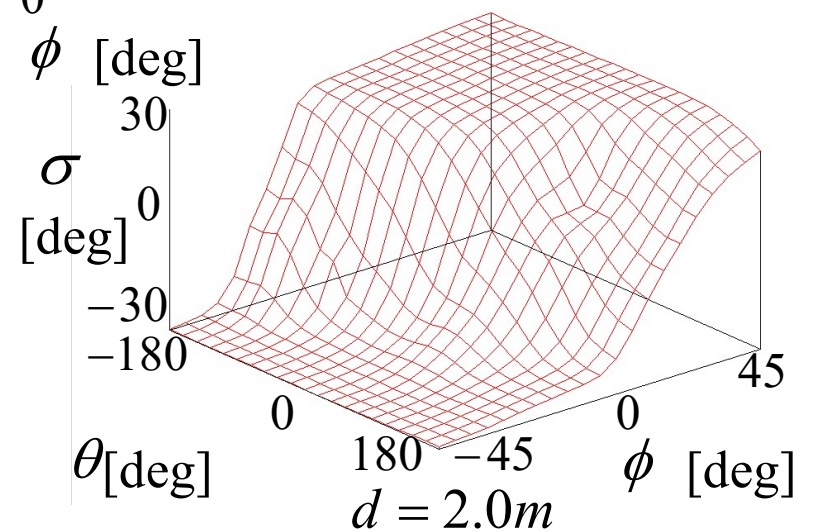
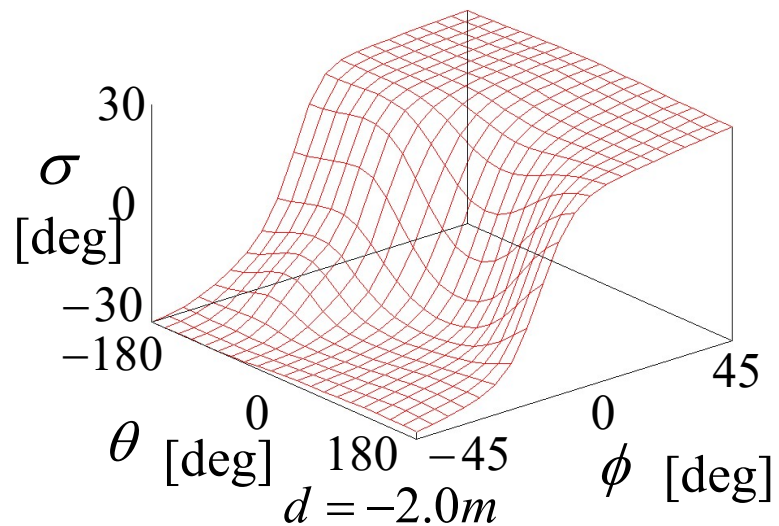
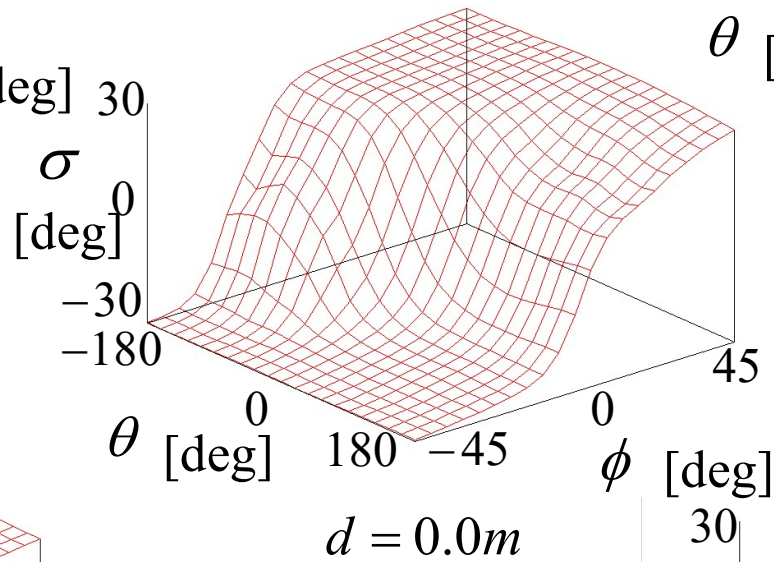
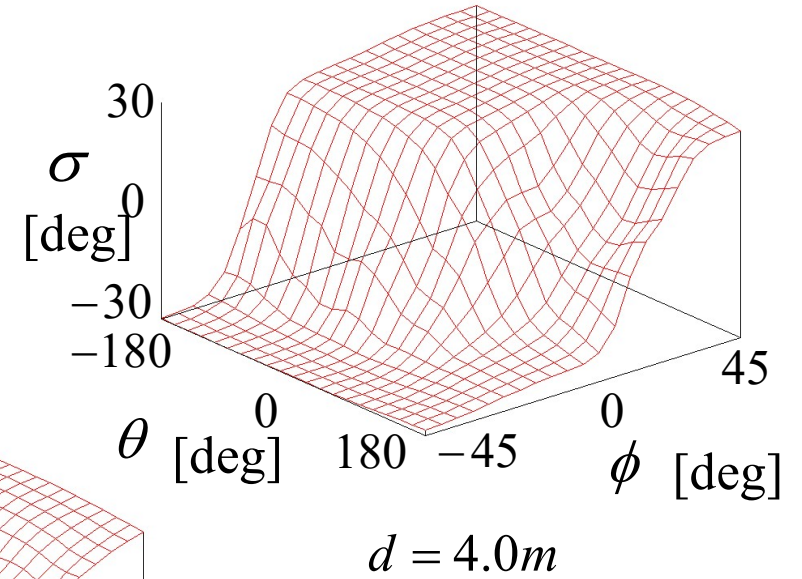
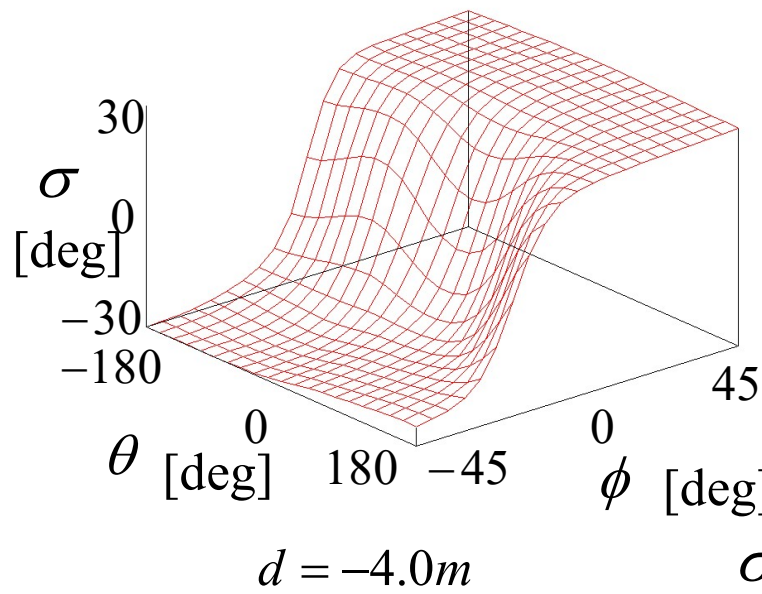
$$-45^{\circ} \leq \phi \leq 45^{\circ}$$

$$-4.0m \leq d \leq 4.0m$$

$$-30^{\circ} \leq \sigma \leq 30^{\circ}$$

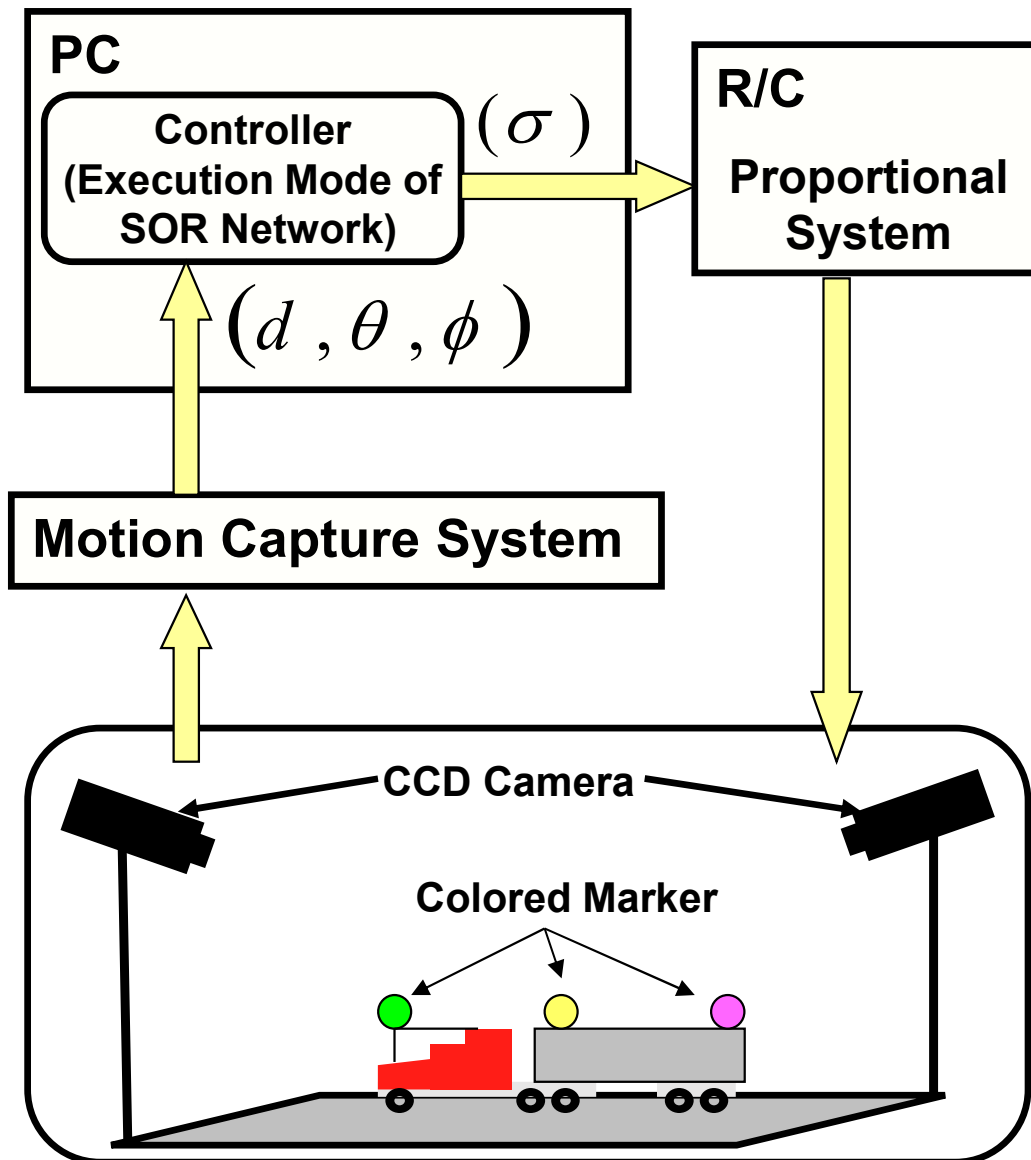


Desirable I/O Relationship Obtained by Learning of SOR Network



Practical Experiment

Overview of the Experiment System



An Example of the Practical Experiment



Initial State of the Trailer-Truck

$$\theta = 135^\circ \quad \phi = 0^\circ \quad d = 2.5m$$



A black Labrador and a yellow Labrador are sitting in a wooden crate. The black dog is on the left, looking directly at the camera. The yellow dog is on the right, looking slightly to the right. Several small, light-colored puppies are visible between them and around them. The crate is made of wooden planks and contains some bedding material.

**Thank You Very Much
For Your Kind Attentions**

Questions are welcome

2019/05/29